Domain Conditions and Social Rationality
Abstract

Satish K. Jain

The problem of aggregating individual preferences arises in contexts in which the decision depends, at least partly, on the preferences of more than one individual. Most important of such contexts include elections and decision-making by representative bodies and committees. For combining individual preferences into overall (social) preferences a variety of procedures are used. The most important of these procedures is that of the method of majority decision (majority rule) under which, between any two alternatives, one alternative is socially at least as good as the other if and only if (iff) the number of individuals who consider the former to be better than the latter is at least as large as the number of individuals who consider the latter to be better than the former (This implies that under the majority rule, between any two alternatives, one alternative is socially better than the other iff the number of individuals who consider the former to be better than the latter is greater than the the number of individuals who consider the latter to be better than the former. Majority rule is one of the simplest and most commonly used procedures for making decisions at a collective level on the basis of preferences of individual members.

Although the appeal of the majority rule is straightforward, the rule has a serious shortcoming as it can give rise to quite paradoxical social preferences. Consider for instance a society consisting of three individuals, A, B, and C, which is to choose one alternative out of the three mutually exclusive alternatives x, y, and z. Let the rankings[1] of these three alternatives in the descending order by the three individuals be:

A's ranking	B's ranking	C's ranking
x	y	z
y	z	x
z	x	y

[1] A preference relation is a ranking or ordering iff it satisfies the properties of reflexivity, connectedness, and transitivity. Reflexivity holds iff every alternative is at least as good as itself; connectedness holds iff between any two distinct alternatives x and y, x is at least as good as y or y is at least as good as x; transitivity holds iff for any three alternatives x, y, z, if x is at least as good as y, and y is at least as good as z, then x is at least as good as z.

As two out of three individuals prefer x to y and only one individual prefers y to x; two out of three individuals prefer y to z and only one individual prefers z to y; and two out of three individuals prefer z to x and only one individual prefers x to z; we obtain the paradoxical result that x is socially better than y, y is socially better than z, and z is socially better than x. Under this scenario it is impossible for the collective to make a rational choice. No matter which of the three alternatives is chosen, among the rejected alternatives there will be an alternative that will be strictly better than the chosen one. This paradox was discovered by the French philosopher and mathematician Marquis de Condorcet[2].

In the above example, although each of the three individuals has a ranking (ordering) of alternatives, the social preferences generated by the majority rule fail to be a ranking. At a first glance, transitivity appears to be an essential requirement of rationality. If transitivity is taken to be a basic requirement then the search for a satisfactory rule for aggregating individual preferences into social preferences must for all practical purposes be confined to rules that aggregate individual orderings into a social ordering. This is the framework in which Arrow formulated the problem of social choice in his seminal work Social Choice and Individual Values[3]. The famous Arrow Impossibility Theorem shows that the following four conditions are logically inconsistent: (i) The rule for aggregating individual orderings into a social ordering, termed social welfare function by Arrow, must work for every logically possible configuration of individual orderings, i.e., the domain of the rule must be unrestricted. (ii) The rule must satisfy the weak Pareto-criterion. The weak Pareto-criterion requires that whenever some alternative x is unanimously preferred by everyone in the society to some other alternative y, x must be socially preferred to y. (iii) The rule must be non-dictatorial. An individual is a dictator iff it is the case that whenever he/she prefers some alternative x to some other alternative y, x is socially preferred to y. The rule is non-dictatorial iff no individual is a dictator. (iv) The rule must be such that the social preferences between any two alternatives must depend only on the individual preferences between those two alternatives. In other words, it must not be the case that although no individual changes his/her preferences between a particular pair of alternatives, the social preferences between them change merely because individuals have changed their preferences among some other alternatives. This requirement is known as the condition of independence of irrelevant alternatives.

It is immediate that no rule that violates the weak Pareto-criterion or is dictatorial

[2]Condorcet, Marquis de. 1785. *Essai sur l'application de l'analyse à la probabilité des décisions rendues à la pluralité des voix.* Paris.

[3]Arrow, Kenneth J., 1951, Social choice and individual values, 2nd ed., 1963, Wiley, New York.

can be acceptable. The condition of independence of irrelevant alternatives is also quite crucial. In its absence the choice procedure becomes ambiguous. Thus, it follows that any rule for aggregating individual preferences into social preferences, that is non-dictatorial and satisfies the weak Pareto-criterion and independence of irrelevant alternatives, must fail to generate a social ranking corresponding to some configuration(s) of individual orderings. The seriousness of the problem of the failure of the rule to generate a social ordering corresponding to every logically possible configuration of orderings of course will differ from rule to rule. Let a rule for aggregating individual orderings into social preferences that are reflexive and connected be termed a social decision rule. Thus from Arrow's Impossibility Theorem it follows that every non-dictatorial social decision rule, satisfying the weak Pareto-criterion and independence of irrelevant alternatives, must fail to generate transitive social preferences corresponding to some configuration(s) of individual orderings. Given a social decision rule, one can partition the set of configurations of individual orderings into two subsets: (i) The configurations for which the rule generates transitive social preferences (ii) The configurations for which the rule does not generate transitive social preferences. If transitivity is considered an essential requirement, then the social decision rule can be used in situations where the configuration of individual orderings belongs to subset (i). Thus, it is quite important to know in the context of social decision rules that are to be used for decision-making the precise composition of subset (i). This, however, is unlikely to be an easily solvable problem for most social decision rules. In fact, excepting the method of majority decision, the precise composition of subset (i) is not known for any social decision rule.

A less intractable problem is as follows. Let f be a social decision rule. Let S be the set of social alternatives and let $\mathcal{T}$ be the set of orderings of S. The set of all nonempty subsets of $\mathcal{T}$ is $2^{\mathcal{T}} - \{\emptyset\}$. Let the set of individuals be N with cardinality n. One can partition the set of all nonempty subsets of $\mathcal{T}$ into two subsets: $(\mathcal{T}_1^T)$ The set of nonempty subsets $\mathcal{D}$ of $\mathcal{T}$ which are such that every profile of individual orderings belonging to $\mathcal{D}^n$ yields transitive social preferences under f. $(\mathcal{T}_2^T)$ The set of nonempty subsets $\mathcal{D}$ of $\mathcal{T}$ which are such that at least one profile of individual orderings belonging to $\mathcal{D}^n$ yields intransitive social preferences under f. A condition on preferences is called a sufficient condition for transitivity if every nonempty $\mathcal{D}$ satisfying the condition belongs to set $(\mathcal{T}_1^T)$. A condition on preferences is called an Inada-type necessary condition for transitivity if every nonempty $\mathcal{D}$ violating the condition belongs to set $(\mathcal{T}_2^T)$.[4] A condition on preferences is called an Inada-type necessary and sufficient condition for transitivity

[4]An Inada-type necessary condition is not a necessary condition in the sense of logic. The expression 'Inada-type necessary condition for transitivity' will be used only as a shorthand expression for the property mentioned above. A similar remark applies to Inada-type necessary conditions for rationality conditions weaker than transitivity.

if every nonempty $\mathcal{D}$ satisfying the condition belongs to set ($\mathcal{T}_1^T$) and every nonempty $\mathcal{D}$ violating the condition belongs to set ($\mathcal{T}_2^T$). Sufficient conditions, Inada-type necessary conditions, and Inada-type necessary and sufficient conditions for other rationality conditions are defined analogously. As an illustration consider the method of majority decision f defined for a set $S = \{x, y, z\}$ of three alternatives and a set $N = \{1, 2, 3\}$ of three individuals. There are thirteen logically possible orderings of three alternatives x, y, z. There are $2^{13} - 1$ nonempty subsets of the set of these 13 orderings. If $\mathcal{D} = \{xyz, (xyz)\}$ then it is immediate that under the method of majority decision every profile of individual orderings belonging to $\mathcal{D}^3$ yields transitive social preferences. Thus, for the method of majority decision defined for $S = \{x, y, z\}$ and $N = \{1, 2, 3\}$, $\{xyz, (xyz)\}$ belongs to set $\mathcal{T}_1^T$. On the other hand we have seen that if $\mathcal{D} = \{xyz, yzx, zxy\}$ and $n = 3$ then there exists a profile of individual orderings that results in intransitive social preferences under the method of majority decision. Thus, for the method of majority decision defined for $S = \{x, y, z\}$ and $N = \{1, 2, 3\}$, $\{xyz, yzx, zxy\}$ belongs to set $\mathcal{T}_2^T$.

In choosing rationally, one chooses an alternative that is best in the sense of being at least as good as every alternative. There can be more than one best alternatives. From the perspective of rational choice it does not matter which of the best alternatives is selected. If the number of alternatives is positive and finite then, in case there is a ranking of all alternatives, a best element will clearly exist. If there is a ranking of all alternatives, one can arrange them from top to bottom in the descending order, and choose the topmost alternative, and in case there are more than one alternatives at the top then choose any one of them. A preference relation is a ranking iff it satisfies the three properties of reflexivity, connectedness, and transitivity. If any one of these three conditions is violated then a best alternative may not exist. We have already seen that if transitivity is violated then it is possible that a best alternative may not exist, as is the case with the Condorcet paradox example. If alternative x is not at least as good as itself, then clearly x cannot be at least as good as every alternative. Thus violation of reflexivity can also result in the non-existence of a best alternative. The reflexivity requirement, although formally needed, is a trivial requirement as the question of some alternative not being at least as good as itself does not arise. Violation of connectedness implies that, for some distinct alternatives x and y, it is the case that neither x is at least as good as y nor y is at least as good as x, i.e., x and y are non-comparable. It is clear that such non-comparability can lead to non-existence of a best alternative. While violation of any of these three conditions can lead to non-existence of a best alternative, none of these three conditions is a necessary condition for the existence of a best alternative. In fact, it is possible for a preference relation to violate all three conditions and still have a best alternative. Thus, for a preference relation defined over a nonempty finite set, being an ordering is sufficient for the existence of a best alternative, but not necessary. A condition weaker than transitivity, called quasi-transitivity, is also sufficient to ensure the existence of a best element in the case of a reflexive and connected preference relation defined over a nonempty finite set. A preference relation is quasi-transitive iff 'better than' relation is transitive. If 'at least as good as' relation is transitive then both 'better than' relation and 'indifferent to' are transitive. In the case of a quasi-transitive relation the relation 'indifferent to' need not be transitive. Both transitivity and quasi-transitivity are conditions defined over triples of alternatives. If these conditions hold for all triples then they hold over all subsets. There is a condition that is weaker than even quasi-transitivity, called acyclicity, that is also sufficient to ensure the existence of a best element in the case of a reflexive and connected preference relation defined over a nonempty finite set. Acyclicity requires that if there is a chain of alternatives connected by 'better than' relation then the first alternative in the chain must be at least good as the last alternative in the chain. Thus, if acyclicity holds, and we have x_1 better than x_2, x_2 better than x_3, $\ldots$, x_{m-1} better than x_m, then it must be the case that x_1 is at least as good as x_m. Acyclicity, unlike quasi-transitivity, is not a condition defined over triples only. Acyclicity holding over all triples does not imply that it would hold over all subsets.

It is possible to argue that the transitivity requirement for social preferences is unnecessarily restrictive. What is needed is that the society or the collective should be able to choose an alternative that is best. Thus, in the context of social decision rules which do not invariably yield quasi-transitive (acyclic) social preferences, it is of considerable importance to characterize those nonempty subsets $\mathcal{D}$ of $\mathcal{T}$ which are such that every profile of individual orderings belonging to $\mathcal{D}^n$ gives rise to quasi-transitive (acyclic) social preferences. In other words, in the context of non-dictatorial social decision rules satisfying the weak Pareto-criterion and independence of irrelevant alternatives, an important social choice-theoretic problem is that of derivation of Inada-type necessary and sufficient conditions for quasi-transitivity (acyclicity).

The pioneering contributions on the conditions for transitivity and quasi-transitivity under the majority rule came from Black (1948), Arrow (1951, 1963), Inada (1964, 1969), and Sen and Pattanaik (1969), among others. Although the majority rule for obvious reasons has attracted the maximum attention[5], other important rules have also been

[5]The literature on the conditions that ensure rationality of social preferences under the majority rule is vast. See Arrow op. cit.; Black, D., 1948, On the rationale of group decision making, The Journal of Political Economy 56, 23-34; Fishburn, Peter C., 1973, The theory of social choice, Princeton University Press, Princeton; Gaertner, Wulf, 1988, Binary inversions and transitive majorities, in Measurement in economics, ed. Wolfgang Eichhorn, 253-267, Springer-Verlag, Berlin; Gaertner, Wulf, 2001, Domain conditions in social choice theory, Cambridge University Press, London; Inada, Ken-ichi, 1964, A note on

analyzed from the perspective of conditions under which they yield rational social preferences[6]. This monograph is almost exclusively concerned with the Inada-type necessary and sufficient conditions for transitivity and quasi-transitivity under the various social decision rules and classes of social decision rules. The social decision rules and the classes of social decision rules for which the domain conditions for transitivity and quasi-transitivity have been discussed in this text include the method of majority decision, the strict majority rule, the class of semi-strict majority rules, the class of special majority rules, the class of non-minority rules, the class of Pareto-inclusive non-minority rules, the class of simple game social decision rules, and the class of neutral and monotonic binary social decision rules (Chapters 3-10). Chapter 2 contains the basic social choice theoretic concepts, definitions, propositions, and theorems that are required for the subject matter of the monograph. In Chapters 3-10 domain conditions are discussed under the assumption that every individual has an ordering of social alternatives. Chapter 11 explores how some

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the simple majority decision rule, Econometrica 32, 525-531; Inada, Ken-ichi, 1969, The simple majority decision rule, Econometrica 37, 490-506; Jain, Satish K., 1985, A direct proof of Inada-Sen-Pattanaik theorem on majority rule, The Economic Studies Quarterly 36, 209-215; Jain, Satish K., 2009, The method of majority decision and rationality conditions, in Ethics, welfare, and measurement, Volume 1 of Arguments for a better world: Essays in honor of Amartya Sen, ed. Kaushik Basu and Ravi Kanbur, 167-192, Oxford University Press, New York; Kelly, J.S., 1974, Necessity conditions in voting theory, Journal of Economic Theory 8, 149-160; Nicholson, Michael B, 1965, Conditions for the 'voting paradox' in committee decisions, Metroeconomica 7, 29-44; Pattanaik, Prasanta K., 1971, Voting and collective choice, Cambridge University Press, Cambridge; Saposnik, Rubin, 1975, On the transitivity of the social preference relation under simple majority rule, Journal of Economic Theory 10, 1-7; Sen, Amartya K., 1966, A possibility theorem on majority decisions, Econometrica 34, 491-499; Sen, Amartya K., 1970, Collective choice and social welfare, Holden-Day, San Francisco; Sen, Amartya K. and Prasanta K. Pattanaik, 1969, Necessary and sufficient conditions for rational choice under majority decision, Journal of Economic Theory 1, 178-202; Slutsky, Steven M., 1977, A characterisation of societies with consistent majority decision, Review of Economic Studies 44, 211-225. The list is by no means exhaustive. For a survey of the literature and an extensive bibliography thereof see Gaertner (2001), op. cit.

[6]See, among others, Batra, Raveendra and Prasanta K. Pattanaik, 1971, Transitivity of social decisions under some more general group decision rules than the method of majority voting, Review of Economic Studies 38, 295-306; Batra, Raveendra and Prasanta K. Pattanaik, 1972, Transitive multi-stage majority decisions with quasi-transitive individual preferences, Econometrica 40, 1121-1135; Dummett, Michael and Robin Farquharson, 1961, Stability in voting, Econometrica 29, 33-43; Fine, Kit, 1973, Conditions for the existence of cycles under majority and non-minority rules, Econometrica 41, 888-899; Fishburn (1973) op. cit.; Jain, Satish K, 1983, Necessary and sufficient conditions for quasi-transitivity and transitivity of special majority rules, Keio Economic Studies 20, 55-63; Jain, Satish K., 1984, Non-minority rules: Characterization of configurations with rational social preferences, Keio Economic Studies 21, 45-54; Jain, Satish K., 1986, Special majority rules: Necessary and sufficient condition for quasi-transitivity with quasi-transitive individual preferences, Social Choice and Welfare 3, 99-106; Jain, Satish K., 1987, Maximal conditions for transitivity under neutral and monotonic binary social decision rules, The Economic Studies Quarterly 38, 124-130; Murakami, Yasusuke, 1968, Logic and social choice, Routledge & Kegan Paul, London; Pattanaik (1971), op. cit,, and Sen (1970), op. cit.

of the results obtained in Chapters 3-10 change when it is assumed that every individual has a reflexive, connected and quasi-transitive weak preference relation over the set of social alternatives.

For the method of majority decision, complete results exist on the domain conditions for transitivity and quasi-transitivity. We have the following: (i) When the number of individuals is even and greater than or equal to 2, an Inada-type necessary and sufficient condition for transitivity is that the extremal restriction holds over every triple of alternatives. (ii) When the number of individuals is odd and greater than or equal to 3, an Inada-type necessary and sufficient condition for transitivity is that the weak Latin Square partial agreement holds over every triple of alternatives. (iii) When the number of individuals is greater than or equal to 5, an Inada-type necessary and sufficient condition for quasi-transitivity is that the Latin Square partial agreement holds over every triple of alternatives. (iv) When the number of individuals is 4, an Inada-type necessary and sufficient condition for quasi-transitivity is that the weak extremal restriction holds over every triple of alternatives. (v) When the number of individuals is 3, an Inada-type necessary and sufficient condition for quasi-transitivity is that the Latin Square linear ordering restriction holds over every triple of alternatives.

For the class of social decision rules that are simple games complete results have been derived on the domain conditions for transitivity and quasi-transitivity. We have the following: (i) A simple game social decision rule yields transitive social weak preference relation for every logically possible profile of individual orderings iff it is null or dictatorial. (ii) For a non-null non-strong simple game social decision rule an Inada-type necessary and sufficient condition for transitivity is that the condition of Latin Square extremal value restriction holds over every triple of alternatives. (iii) For a non-dictatorial strong simple game social decision rule an Inada-type necessary and sufficient condition for transitivity is that the condition of weak Latin Square extremal value restriction holds over every triple of alternatives. (iv) A simple game social decision rule yields quasi-transitive social weak preference relation for every logically possible profile of individual orderings iff it is null or is such that there is a unique minimal winning coalition. (v) For a non-null simple game social decision rule with at least two minimal winning coalitions an Inada-type necessary and sufficient condition for quasi-transitivity is that the condition of Latin Square unique value restriction holds over every triple of alternatives.

The class of strict majority rules is a subclass of the class of social decision rules that are simple games; consequently, we have complete results on the domain conditions for transitivity and quasi-transitivity for it. These are: (i) For a p-strict majority rule such that there exists a partition (N_1, N_2) of the set of individuals N such that $\#N_1 \leq pn \wedge \#N_2 \leq pn$ an Inada-type necessary and sufficient condition for transitivity is that the condition of Latin Square extremal value restriction holds over every triple of alternatives. (ii) For a p-strict majority rule such that there does not exist a partition (N_1, N_2) of the set of individuals N, $\#N \geq 3$, such that $\#N_1 \leq pn \wedge \#N_2 \leq pn$ an Inada-type necessary and sufficient condition for transitivity is that the condition of weak Latin Square extremal value restriction holds over every triple of alternatives. (iii) If $n = \lfloor pn \rfloor + 1$, then p-strict majority rule yields quasi-transitive social weak preference relation for every logically possible profile of individual orderings. (iv) For a p-strict majority rule, if $n > \lfloor pn \rfloor + 1$, then an Inada-type necessary and sufficient condition for quasi-transitivity is that the condition of Latin Square unique value restriction holds over every triple of alternatives. In view of these results it follows that for the strict majority rule we have: (i) When the number of individuals is even and greater than or equal to 2, an Inada-type necessary and sufficient condition for transitivity is that the condition of Latin Square extremal value restriction holds over every triple of alternatives. (ii) When the number of individuals is odd and greater than or equal to 3, an Inada-type necessary and sufficient condition for transitivity is that the weak Latin Square extremal value restriction holds over every triple of alternatives. (iii) When the number of individuals is greater than or equal to 3, an Inada-type necessary and sufficient condition for quasi-transitivity is that the condition of Latin Square unique value restriction holds over every triple of alternatives.

For the class of special majority rules we have the following: (i) Satisfaction of placement restriction over every triple of alternatives is a sufficient condition for transitivity under every special majority rule. (ii) For every special majority rule, satisfaction of placement restriction over every triple of alternatives is an Inada-type necessary and sufficient condition for transitivity for infinitely many values of n. (iii) Satisfaction of Latin Square partial agreement over every triple of alternatives is a sufficient condition for quasi-transitivity under every special majority rule. (iv) For every special majority rule, there exists a positive integer n' such that the satisfaction of Latin Square partial agreement over every triple of alternatives is an Inada-type necessary and sufficient condition for all $n \geq n'$. Two-thirds majority rule belongs to the class of special majority rules. For the two thirds majority rule we have: (i) If the number of individuals $n \geq 10$, then the satisfaction of placement restriction over every triple of alternatives is an Inada-type necessary and sufficient condition for transitivity. (ii) If the number of individuals $n \geq 10$, then the satisfaction of Latin Square partial agreement over every triple of alternatives is an Inada-type necessary and sufficient condition for quasi-transitivity. From the transitivity result for the two-thirds majority rule it seems that it might be the case that for every special majority rule there exists a positive integer n' such that the satisfaction of placement restriction over every triple of alternatives is an Inada-type necessary and sufficient condition for transitivity for all $n \geq n'$.

The satisfaction over every triple of alternatives of at least one of the three conditions of strict placement restriction, partial agreement, and strongly antagonistic preferences (1) is a sufficient condition for transitivity under every p-semi-strict majority rule. It is also the case that for every p-semi-strict majority rule there exists a positive even integer n' such that for every even integer $n \geq n'$ the satisfaction over every triple of alternatives of at least one of the three conditions of strict placement restriction, partial agreement, and strongly antagonistic preferences (1) is an Inada-type necessary and sufficient condition for transitivity. From the proofs of the concerned theorems it is clear that the transitivity conditions for an odd number of individuals are likely to be weaker than the above ones, as indeed is the case with both the method of majority decision and the strict majority rule. The satisfaction over every triple of alternatives of value restriction (2) or absence of unique extremal value is a sufficient condition for quasi-transitivity under every p-semi-strict majority rule. It is also the case that for every p-semi-strict majority rule there exist infinitely many values of n for which the satisfaction over every triple of alternatives of value restriction (2) or absence of unique extremal value is an Inada-type necessary and sufficient condition for quasi-transitivity.

For the class of Pareto-inclusive strict majority rules we have the following results: (i) If $n = 2$ then the satisfaction over every triple of alternatives of extremal restriction is an Inada-type necessary and sufficient condition for transitivity under every Pareto-inclusive strict majority rule. (ii) For Pareto-inclusive p-strict majority rule, if $n \geq 3$ and $n = \lfloor pn \rfloor + 1$, then the satisfaction over every triple of alternatives of at least one of the three conditions of placement restriction, absence of unique extremal value, and strongly antagonistic preferences (2) is an Inada-type necessary and sufficient condition for transitivity. (iii) The satisfaction over every triple of alternatives of placement restriction or absence of unique extremal value is a sufficient condition for transitivity under every Pareto-inclusive strict majority rule. (iv) For the class of Pareto-inclusive strict majority rules, the satisfaction over every triple of alternatives of placement restriction or absence of unique extremal value is maximally sufficient for transitivity. (v) If $n = \lfloor pn \rfloor + 1$, then Pareto-inclusive p-strict majority rule yields quasi-transitive weak preference relation for every logically possible profile of individual orderings. (vi) If $n = \lfloor pn \rfloor + 2$, then the satisfaction over every triple of alternatives of Latin Square unique value restriction is an Inada-type necessary and sufficient condition for quasi-transitivity under Pareto-inclusive p-strict majority rule. (vii) If $n > \lfloor pn \rfloor + 2$, then the satisfaction over every triple of alternatives of Latin Square unique value restriction or limited agreement is an Inada-type necessary and sufficient condition for quasi-transitivity under Pareto-inclusive p-strict majority rule.

For the class of neutral and monotonic binary social decision rules we have the following results: (i) The satisfaction over every triple of alternatives of strict placement restriction is a sufficient condition for transitivity under every neutral and monotonic binary social decision rule. (ii) For the class of neutral and monotonic binary social decision rules, the satisfaction over every triple of alternatives of strict placement restriction is maximally sufficient for transitivity. (iii) The satisfaction over every triple of alternatives of value restriction (2) is a sufficient condition for quasi-transitivity under every neutral and monotonic binary social decision rule.

For the class of neutral and monotonic binary social decision rules satisfying the Pareto-criterion we have the following results: (i) The satisfaction over every triple of alternatives of placement restriction is a sufficient condition for transitivity under every neutral and monotonic binary social decision rule satisfying the Pareto-criterion. (ii) For the class of neutral and monotonic binary social decision rules satisfying the Pareto-criterion, the satisfaction over every triple of alternatives of placement restriction is maximally sufficient for transitivity. (iii) The satisfaction over every triple of alternatives of value restriction (2) or limited agreement is a sufficient condition for quasi-transitivity under every neutral and monotonic binary social decision rule satisfying the Pareto-criterion.

All the above results have been derived under the assumption that every individual has an ordering over the set of social alternatives. Statements of domain conditions for the class of neutral and monotonic binary social decision rules become extremely simple if it is assumed that every individual has a linear ordering of social alternatives. We have the following results when individuals have linear orderings over the set of social alternatives: (i) A neutral and monotonic binary social decision rule yields transitive social weak preference relation for every logically possible profile of individual linear orderings iff it is null or dictatorial. (ii) For a non-dictatorial neutral and monotonic binary social decision rule which is such that for every partition (N_1, N_2) of the set of individuals N, N_1 or N_2 is a decisive set, an Inada-type necessary and sufficient condition for transitivity is that there be no Latin Square over any triple of alternatives. (iii) For a non-null neutral and monotonic binary social decision rule which is such that there exists a partition (N_1, N_2) of the set of individuals N such that neither N_1 nor N_2 is a decisive set, an Inada-type necessary and sufficient condition for transitivity is that over every triple of alternatives there be at most one ordering belonging to any particular Latin Square. (iv) A neutral and monotonic binary social decision rule yields quasi-transitive social weak preference relation for every logically possible profile of individual linear orderings iff there is at most one minimal decisive set. (v) For a neutral and monotonic binary social decision rule which is such that there are at least two minimal decisive sets, an Inada-type necessary and sufficient condition for quasi-transitivity is that there be no Latin Square over any triple of alternatives.

Some results are available for the case of when individual weak preference relations are reflexive, connected and quasi-transitive. For the method of majority decision we have: (i) When the number of individuals is greater than or equal to 5, an Inada-type necessary and sufficient condition for quasi-transitivity is that the Latin Square partial agreement - Q holds over every triple of alternatives. (ii) When the number of individuals is 4, an Inada-type necessary and sufficient condition for quasi-transitivity is that the weak extremal restriction - Q holds over every triple of alternatives. (iii) When the number of individuals is 3, an Inada-type necessary and sufficient condition for quasi-transitivity is that the Latin Square linear ordering restriction - Q holds over every triple of alternatives. (iv) When the number of individuals is 2, an Inada-type necessary and sufficient condition for quasi-transitivity is that the Latin Square intransitive relation restriction - Q holds over every triple of alternatives. For quasi-transitivity under the class of social decision rules that are simple games we have the same results as in the case of individuals having orderings over social alternatives. For the class of neutral and monotonic binary social decision rules, satisfaction of VR (2) over every triple of alternatives continues to be sufficient for quasi-transitivity.

In addition to the results on the domain conditions, the monograph also contains characterizations for the following decision rules and the classes of social decision rules: The strict majority rule, the class of strict majority rules, the class of social decision rule that are simple games, the class of social decision rules that are strong simple games, and the class of neutral and monotonic binary social decision rules.